COMP 532
Machine Learning and BioInspired Optimization

Lecture 4 Reinforcement Learning

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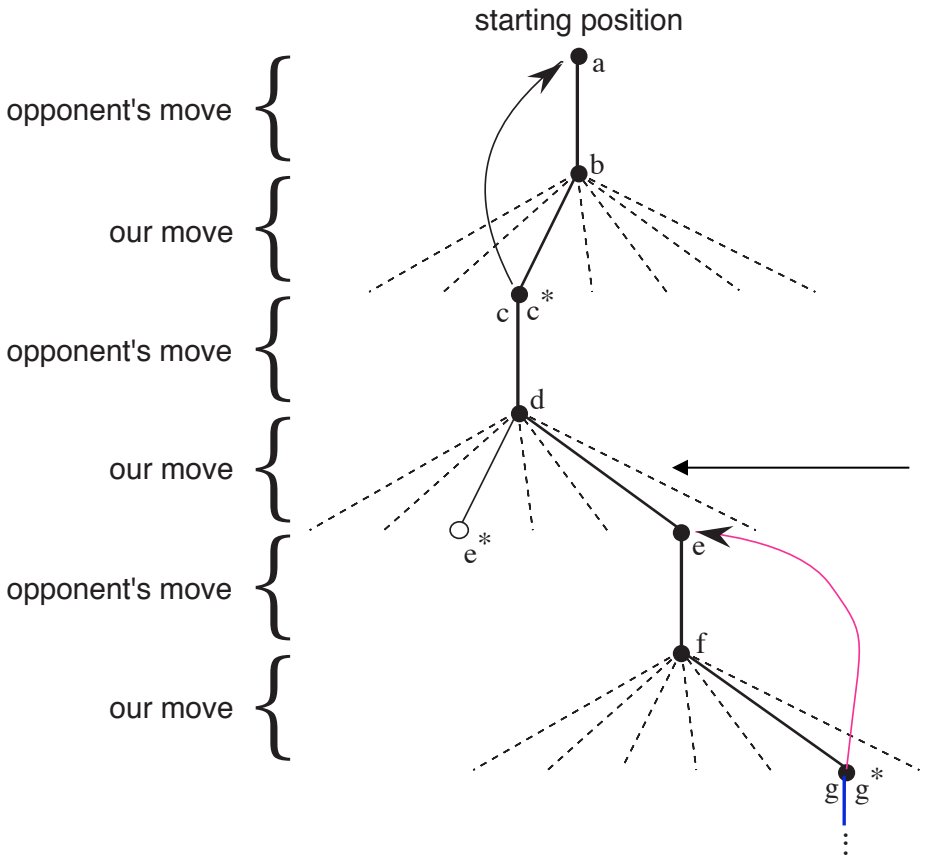
The Book

- Part I: The Problem
  - Introduction
  - Evaluative Feedback
  - The Reinforcement Learning Problem
- Part II: Elementary Solution Methods
  - Dynamic Programming
  - Monte Carlo Methods
  - Temporal Difference Learning
- Part III: A Unified View
  - Eligibility Traces
  - Generalization and Function Approximation
  - Planning and Learning
  - Dimensions of Reinforcement Learning
  - Case Studies

RL Learning Rule for Tic-Tac-Toe



"Exploratory" move

s – the state before our greedy move

s' – the state after our greedy move

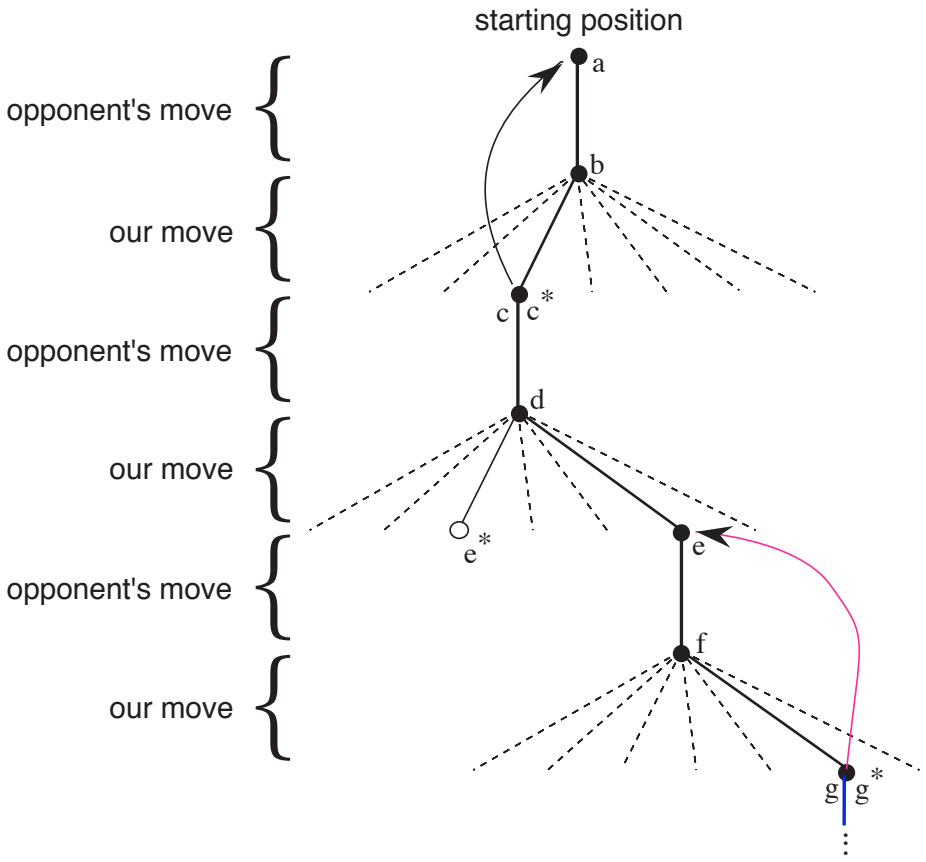
We increment each $V(s)$ toward $V(s')$ – a ***backup***:

$$V(s) \leftarrow V(s) + \alpha\left[V(s') - V(s)\right]$$

a small positive fraction, e.g., $\alpha = .1$

the ***step - size parameter***

RL Learning Rule for Tic-Tac-Toe



$$V(a) \leftarrow V(a) + \alpha[V(c) - V(a)]$$

$$V(e) \leftarrow V(e) + \alpha[V(g) - V(e)]$$

s – the state before our greedy move

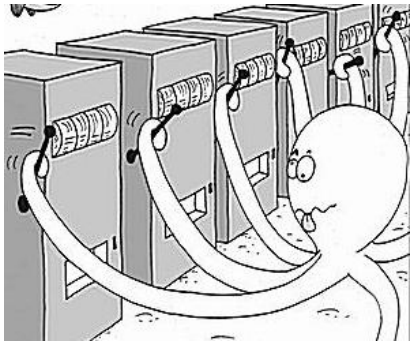
s' – the state after our greedy move

We increment each $V(s)$ toward $V(s')$ – a ***backup***:

$$V(s) \leftarrow V(s) + \alpha\left[V(s') - V(s)\right]$$

Evaluative Feedback

- **Evaluating** actions vs. **instructing** by giving correct actions
- Pure evaluative feedback depends totally on the action taken.
  Pure instructive feedback depends not at all on the action taken.
- Supervised learning is instructive; optimization is evaluative
- **Associative** vs. **Nonassociative**:
  - Associative: inputs mapped to outputs;
    learn the best output **for each** input
  - Nonassociative: "learn" (find) one best output
- *n*-armed bandit (at least how we treat it) is:
  - Nonassociative
  - Evaluative feedback



The *n*-Armed Bandit Problem

- Choose repeatedly from one of *n* actions; each choice is called a **play**
- After each play a_t, you get a reward r_t, where

$$E\{r_t \mid a_t\} = Q^*(a_t)$$

These are unknown ***action values***
Distribution of r_t depends only on a_t

- Objective is to maximize the reward in the long term, e.g., over 1000 plays

To solve the *n*-armed bandit problem,
you must **explore** a variety of actions
and **exploit** the best of them

The Exploration/Exploitation Dilemma

- Suppose you form estimates

$$Q_t(a) \approx Q^*(a)$$ **action value estimates**

- The **greedy** action at *t* is a^*_t

$$a_t^* = \arg\max_a Q_t(a)$$

$$a_t = a_t^* \Rightarrow \text{exploitation}$$

$$a_t \neq a_t^* \Rightarrow \text{exploration}$$

- You can't exploit all the time; you can't explore all the time
- You can never stop exploring; but you should always reduce exploring. Maybe.

Action-Value Methods

- Methods that adapt action-value estimates and nothing else, e.g.: suppose by the *t*-th play, action *a* had been chosen k_a times, producing rewards $r_1, r_2, \ldots, r_{k_a}$, then

$$Q_t(a) = \frac{r_1 + r_2 + \cdots + r_{k_a}}{k_a}$$

"sample average"

- $\lim_{k_a \to \infty} Q_t(a) = Q^*(a)$

ε-Greedy Action Selection

- Greedy action selection:

$$a_t = a_t^* = \arg\max_a Q_t(a)$$

- ε-Greedy:

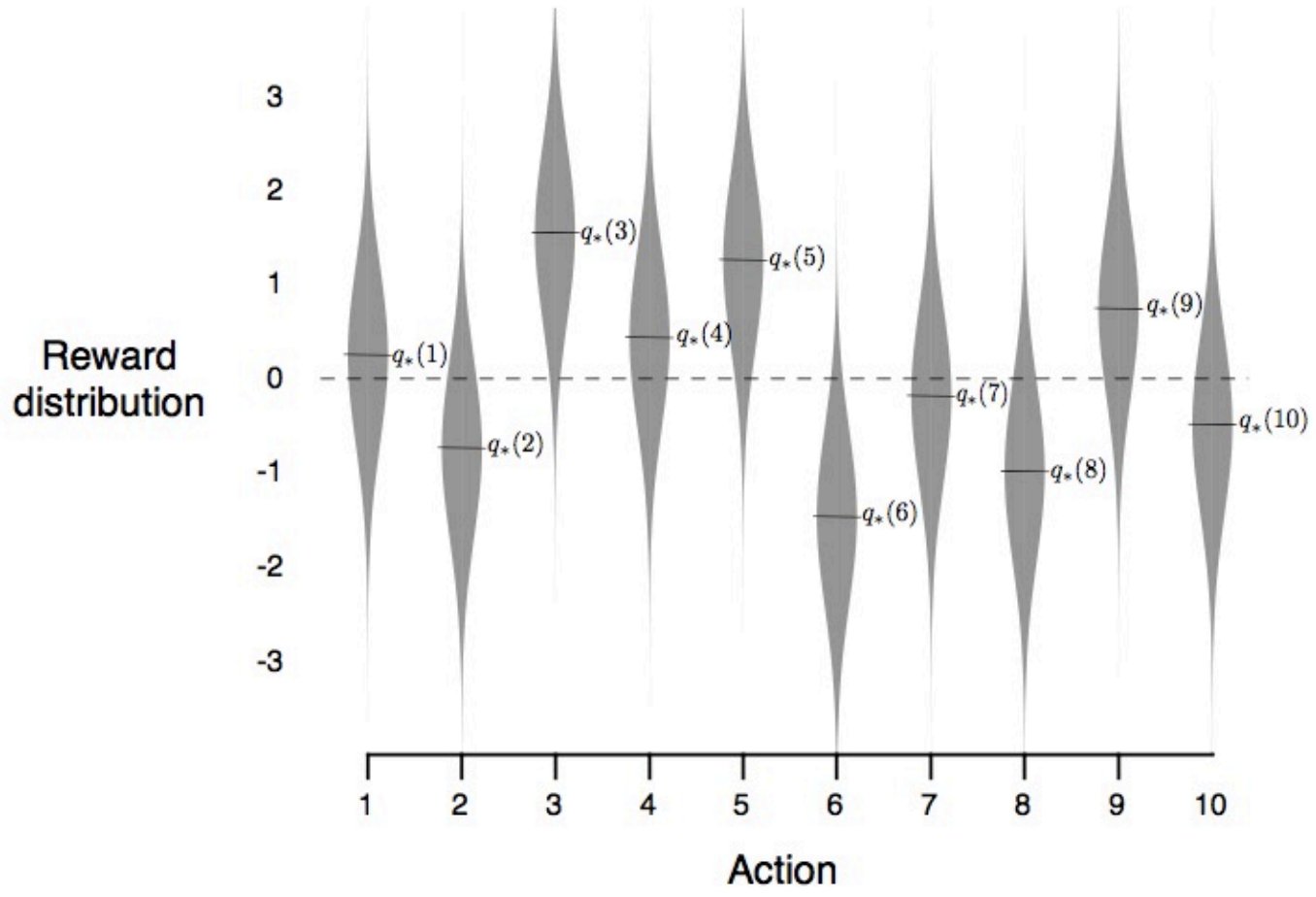
$$a_t = \begin{cases} a_t^* & \text{with probability } 1 - \varepsilon \\ \text{random action with probability} & \varepsilon \end{cases}$$

. . . the simplest way to balance exploration and exploitation

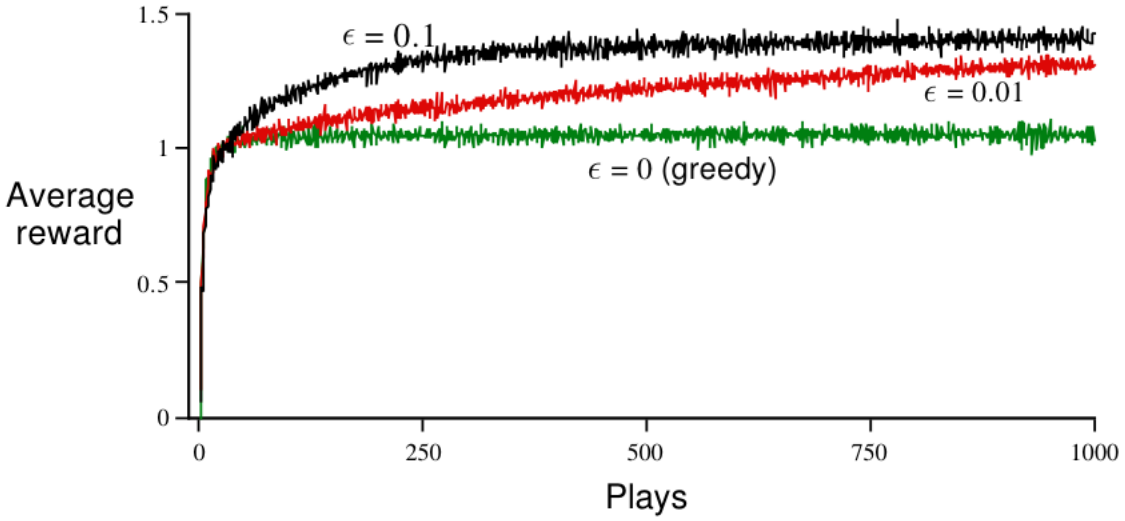
10-Armed Testbed

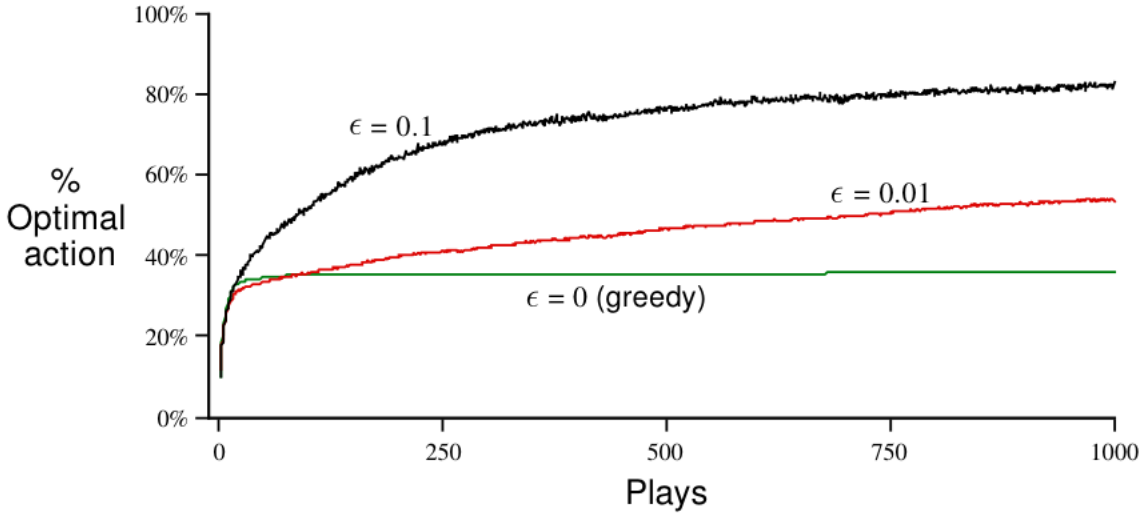
- *n* = 10 possible actions
- Each $Q^*(a)$ is chosen randomly from a normal distrib.: $\eta(0,1)$
- each r_t is also normal: $\eta(Q^*(a_t),1)$
- 1000 plays
- repeat the whole thing 2000 times and average the results

10-Armed Testbed



ε-Greedy Methods on the 10-Armed Testbed





Softmax Action Selection

- Softmax action selection methods define action probabilities by estimated values.
- The most common softmax uses a Gibbs, or Boltzmann, distribution:

Choose action a on play t with probability

$$\frac{e^{Q_t(a)/\tau}}{\sum_{b=1}^{n} e^{Q_t(b)/\tau}}$$

where τ is the

"computational temperature"

Incremental Implementation

Recall the sample average estimation method:

The average of the first *k* rewards is (dropping the dependence on a):

$$Q_k = \frac{r_1 + r_2 + \cdots r_k}{k}$$

Can we do this incrementally (without storing all the rewards)?

We could keep a running sum and count, or, equivalently:

$$Q_{k+1} = Q_k + \frac{1}{k+1}\left[r_{k+1} - Q_k\right]$$

This is a common form for update rules:

NewEstimate = *OldEstimate* + *StepSize*[*Target* – *OldEstimate*]

Tracking a Nonstationary Problem

Choosing Q_k to be a sample average is appropriate in a stationary problem

(i.e., when none of the $Q^*(a)$ change over time)

But not in a nonstationary problem.

Better in the nonstationary case is:

$$Q_{k+1} = Q_k + \alpha \left[r_{k+1} - Q_k \right]$$

for *constant* α, $0 < \alpha \leq 1$

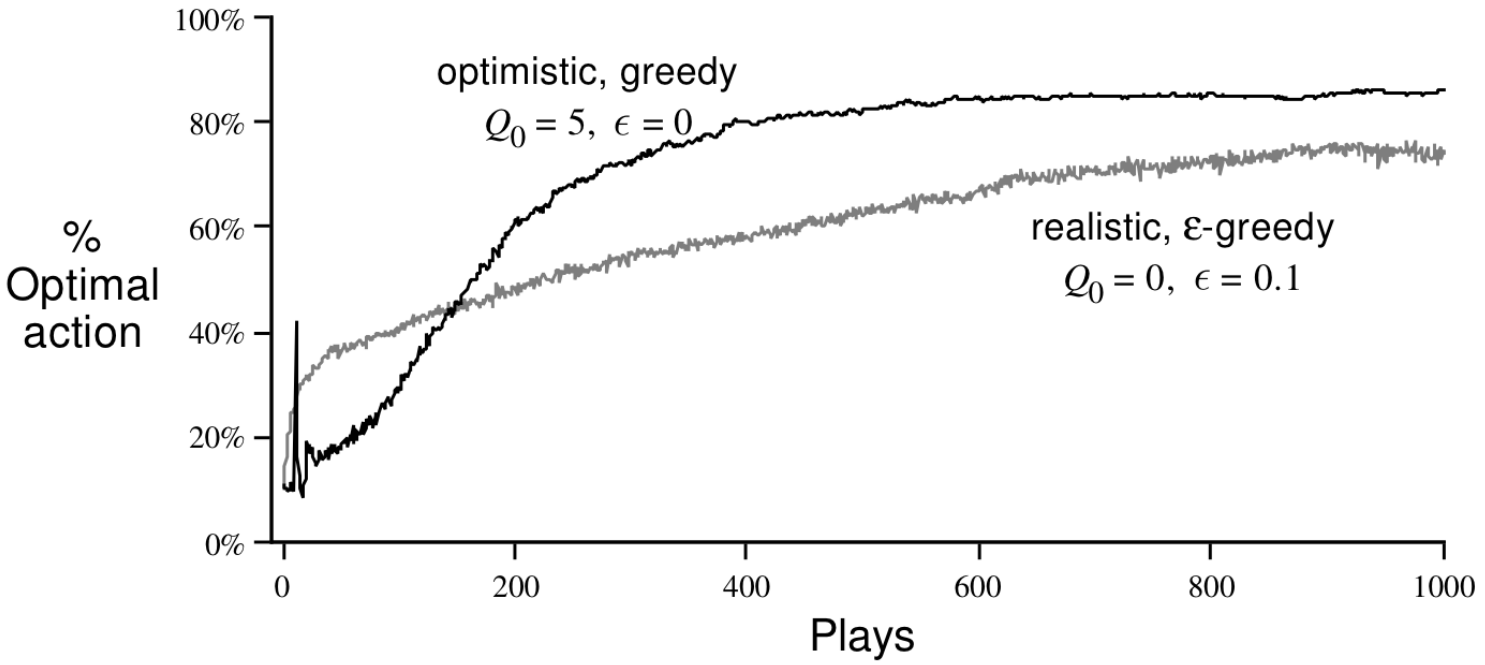
$$= (1-\alpha)^k Q_0 + \sum_{i=1}^{k} \alpha (1-\alpha)^{k-i} r_i$$

exponential, recency-weighted average

Optimistic Initial Values

- All methods so far depend on $Q_0(a)$, i.e., they are **biased.**
- Suppose instead we initialize the action values optimistically,

  i.e., on the 10-armed testbed, use $Q_0(a) = 5 \quad \text{for all } a$



Conclusions

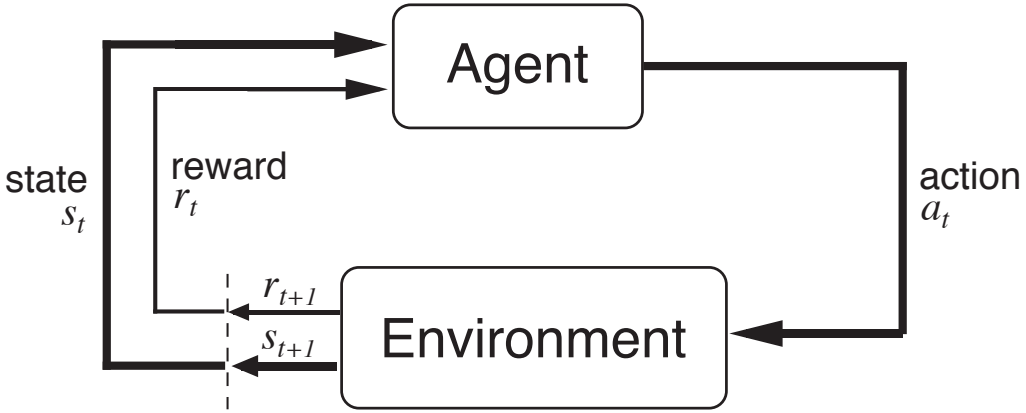
- These are all very simple methods
  - but they are complicated enough—we will build on them
  - we should understand them *completely*

The Reinforcement Learning Problem

Markov Decision Processes and Value Functions

- describe the RL problem we will be studying for the remainder of this part of the course
- present idealized form of the RL problem for which we have precise theoretical results;
- introduce key components of the mathematics: value functions and Bellman equations;

The Agent-Environment Interface



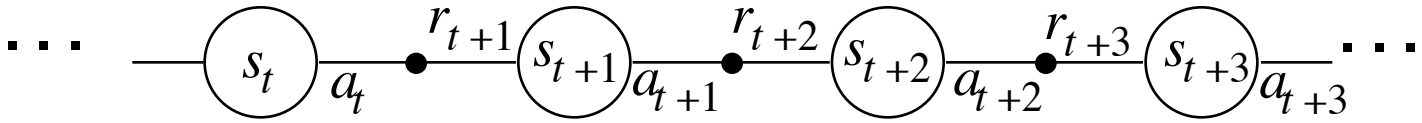
Agent and environment interact at discrete time steps : $t = 0, 1, 2, \mathrm{K}$

Agent observes state at step t: $s_t \in S$

produces action at step t: $a_t \in A(s_t)$

gets resulting reward : $r_{t+1} \in \Re$

and resulting next state : s_{t+1}



The Agent Learns a Policy

Policy at step t, π_t :

a mapping from states to action probabilities

$\pi_t(s,a) =$ probability that $a_t = a$ when $s_t = s$

- Reinforcement learning methods specify how the agent changes its policy as a result of experience.
- Roughly, the agent's goal is to get as much reward as it can over the long run.

Getting the Degree of Abstraction Right

- Time steps need not refer to fixed intervals of real time.
- Actions can be low level (e.g., voltages to motors), or high level (e.g., accept a job offer), “mental” (e.g., shift in focus of attention), etc.
- States can be low-level “sensations”, or they can be abstract, symbolic, based on memory, or subjective (e.g., the state of being “surprised” or “lost”).
- An RL agent is not like a *whole* animal or robot.
- Reward computation is in the agent’s environment because the agent cannot change it arbitrarily.
- The environment is not necessarily unknown to the agent, only incompletely controllable.

Goals and Rewards

- Is a scalar reward signal an adequate notion of a goal?—maybe not, but it is surprisingly flexible.
- A goal should specify **what** we want to achieve, not **how** we want to achieve it.
- A goal must be outside the agent's direct control—thus outside the agent.
- The agent must be able to measure success:
  - explicitly;
  - frequently during its lifespan.

The reward hypothesis

- That all of what we mean by goals and purposes can be well thought of as the maximization of the cumulative sum of a received scalar signal (reward)

Returns

Suppose the sequence of rewards after step t is:

$$r_{t+1}, r_{t+2}, r_{t+3}, \ldots$$

What do we want to maximize?

In general,

we want to maximize the **expected return**, $E\{R_t\}$, for each step t.

Episodic tasks: interaction breaks naturally into episodes, e.g., plays of a game, trips through a maze.

$$R_t = r_{t+1} + r_{t+2} + \ldots + r_T,$$

where T is a final time step at which a **terminal state** is reached, ending an episode.

Returns for Continuing Tasks

Continuing tasks: interaction does not have natural episodes.

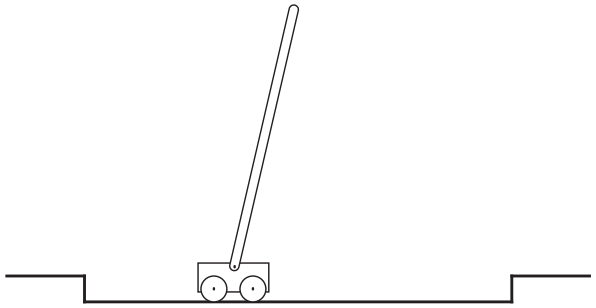
Discounted return:

$$R_t = r_{t+1} + \gamma r_{t+2} + \gamma^2 r_{t+3} + \cdots = \sum_{k=0}^{\infty} \gamma^k r_{t+k+1},$$

where $\gamma, 0 \le \gamma \le 1,$ is the **discount rate**.

shortsighted $0 \leftarrow \gamma \rightarrow 1$ farsighted

An Example



Avoid **failure:** the pole falling beyond a critical angle or the cart hitting end of track.

As an **episodic task** where episode ends upon failure:

$$\text{reward} = +1 \text{ for each step before failure}$$

$$\Rightarrow \text{return} = \text{number of steps before failure}$$

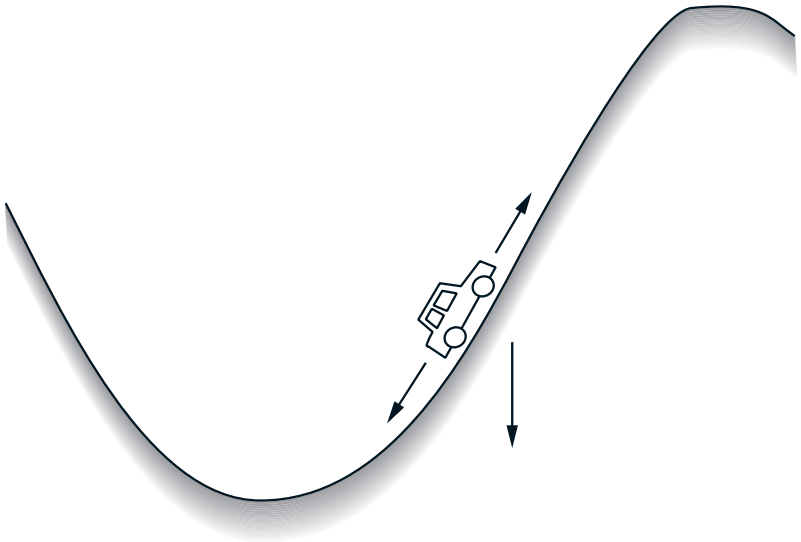
As a **continuing task** with discounted return:

$$\text{reward} = -1 \text{ upon failure; } 0 \text{ otherwise}$$

$$\Rightarrow \text{return} = -\gamma^k, \text{ for } k \text{ steps before failure}$$

In either case, return is maximized by avoiding failure for as long as possible.

Another Example



Get to the top of the hill
as quickly as possible.

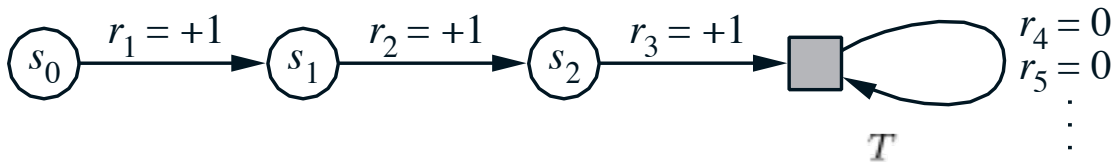
reward = –1 for each step where **not** at top of hill

$\Rightarrow$ return = – number of steps before reaching top of hill

Return is maximized by minimizing
number of steps to reach the top of the hill.

A Unified Notation

- In episodic tasks, we number the time steps of each episode starting from zero.
- We usually do not have to distinguish between episodes, so we write s_t instead of $s_{t,j}$ for the state at step *t* of episode *j*.
- Think of each episode as ending in an absorbing state that always produces reward of zero:



- We can cover <u>all</u> cases by writing $R_t = \sum_{k=0}^{T} \gamma^k r_{t+k+1}$,

  including the possibility that $T = \infty$ or $\gamma = 1$ (not both)

The Markov Property

- By "the state" at step *t*, the book means whatever information is available to the agent at step *t* about its environment.
- The state can include immediate "sensations", highly processed sensations, and structures built up over time from sequences of sensations.
- Ideally, a state should summarize past sensations so as to retain all "essential" information, i.e., it should have the **Markov Property**:

$$Pr\{s_{t+1} = s', r_{t+1} = r \mid s_t, a_t, r_t, s_{t-1}, a_{t-1}, \ldots, r_1, s_0, a_0\}$$
$$= Pr\{s_{t+1} = s', r_{t+1} = r \mid s_t, a_t\},$$

for all s' and r, and all histories $s_t, a_t, r_t, s_{t-1}, a_{t-1}, \ldots, r_1, s_0, a_0$

Markov Decision Processes

- If a reinforcement learning task has the Markov Property, it is basically a **Markov Decision Process (MDP)**.
- If state and action sets are finite, it is a **finite MDP**.
- To define a finite MDP, you need to give:
  - **state and action sets**
  - one-step "dynamics" defined by **transition probabilities**:

$$\mathbf{P}^{a}_{ss'} = \Pr\left\{ s_{t+1} = s' \mid s_t = s, a_t = a \right\} \quad \text{for all } s, s' \in S, a \in A(s).$$

  - **reward probabilities**:

$$\mathbf{R}^{a}_{ss'} = E\left\{ r_{t+1} \mid s_t = s, a_t = a, s_{t+1} = s' \right\} \quad \text{for all } s, s' \in S, a \in A(s).$$

An Example Finite MDP

Recycling Robot

- At each step, robot has to decide whether it should (1) actively search for a can, (2) wait for someone to bring it a can, or (3) go to home base and recharge.
- Searching is better but runs down the battery; if runs out of power while searching, has to be rescued (which is bad).
- Decisions made on basis of current energy level: high, low.
- Reward = number of cans collected

Recycling Robot MDP

$S = \{\texttt{high}, \texttt{low}\}$

$A(\texttt{high}) = \{\texttt{search}, \texttt{wait}\}$

$A(\texttt{low}) = \{\texttt{search}, \texttt{wait}, \texttt{recharge}\}$

$\mathbf{R}^{\texttt{search}}$ = expected no. of cans while searching

$\mathbf{R}^{\texttt{wait}}$ = expected no. of cans while waiting

$\mathbf{R}^{\texttt{search}} > \mathbf{R}^{\texttt{wait}}$